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System of Linear Algebraic Equation

Let us concider the system of eqn

$$\left.\begin{aligned} x_1+x_2+x_3&=6 \\ 3x_1+3x_2+4x_3&=20 \\ 2x_1+x_2+3x_3&=13 \end{aligned}\right\}.$$

Let $A = \text{coeff}^{n} = \begin{pmatrix} 1 & 1 & 1 \\ 3 & 3 & 4 \\ 2 & 1 & 3 \end{pmatrix}$

$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ $b = \begin{pmatrix} 6 \\ 20 \\ 13 \end{pmatrix}$

then given system can be written as.

$Ax = b.$

$$\underbrace{[A|b]}_{\boxed{\text{augmented}}} = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 3 & 3 & 4 & 20 \\ 2 & 1 & 3 & 13 \end{array}\right] \begin{array}{l} \\ R_2 - 3R_1 \\ R_3 - 2R_1 \end{array}$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & \boxed{0} & 1 & 2 \\ 0 & \boxed{-1} & 1 & 1 \end{array}\right]$$

Now the system become—

$$x_1 + x_2 + x_3 = 6.$$
$$x_3 = 2$$
$$-x_2 + x_3 = 1$$

Nex elimination

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 0 & 1 & 2 \\ 0 & -1 & 1 & 1 \end{array}\right]$$

Now we apply partial-pivoting

$\longrightarrow R_{32}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$\therefore\ x_1 + x_2 + x_3 = 6$

$0 - x_2 + x_3 = 1$

$x_3 = 2$

Now use backward Substitution.

$x_3 = 2, \quad -x_2 + x_3 = 1$

$\Rightarrow -x_2 = 1 - 2$

$\Rightarrow -x_2 = 2 + 1$

$\Rightarrow -x_2 = 2 - 1$

$\Rightarrow x_2 = 1$

$\therefore\ x_1 + x_2 + x_3 = 6.$

$\therefore\ x_1 + 1 + 2 = 6$

$\therefore\ x_1 = 6 - 3$

$\therefore\ x_1 = 3$

$\therefore$ Required Soln $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}.$

(ii) Complete pivoting

$$\boxed{A|b} = \left[\begin{array}{ccc|c} \boxed{1} & 1 & 1 & 6 \\ 3 & 3 & \boxed{4} & 20 \\ 2 & 1 & 3 & 13 \end{array}\right] \rightarrow C_{31}$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 4 & 3 & 3 & 20 \\ 3 & 1 & 2 & 13 \end{array}\right] \rightarrow R_{21}$$

$$\rightarrow \left[\begin{array}{ccc|c} 4 & 3 & 3 & 20 \\ 1 & 1 & 1 & 6 \\ 3 & 1 & 2 & 13 \end{array}\right] \begin{array}{l} \\ \rightarrow R_2 - \frac{1}{4}R_1 \\ \rightarrow R_3 - \frac{3}{4}R_1 \end{array}$$

$\rightarrow$

$1-\frac{3}{4}$

$1-\frac{9}{4}$

$2-\frac{9}{9}$

$$\rightarrow \left[\begin{array}{ccc|c} 4 & 3 & 3 & 20 \\ 0 & \frac{1}{4} & \frac{1}{4} & 1 \\ 0 & \boxed{-\frac{5}{4}} & -\frac{1}{4} & -2 \end{array}\right] R_{3L}$$

$$\rightarrow \begin{array}{c} \begin{array}{ccc} x_3 & x_2 & x_1 \end{array} \\ \left[\begin{array}{ccc|c} 4 & 3 & 3 & 20 \\ 0 & -\frac{5}{4} & -\frac{1}{4} & -2 \\ 0 & \frac{1}{4} & \frac{1}{4} & 1 \end{array}\right] \end{array} R_3 + \frac{1}{5} R_2$$

$$\rightarrow \begin{array}{c} \begin{array}{ccc} x_3 & x_2 & x_1 \end{array} \\ \left[\begin{array}{ccc|c} 4 & 3 & 3 & 20 \\ 0 & -\frac{5}{4} & -\frac{1}{4} & -2 \\ 0 & 0 & \frac{1}{5} & \frac{3}{5} \end{array}\right] \end{array}$$

$\frac{1}{4}-\frac{1}{20}$ $\quad -8$

$=\frac{5-1}{20}$ $\quad 1-\frac{2}{5}$

$\therefore$ Then the system become-

$$4x_3+3x_2+3x_1=20$$

$$-\frac{5}{4}x_2-\frac{1}{4}x_1=-2$$

$$\frac{1}{5}x_1=\frac{3}{5}$$

$$\therefore \boxed{x_1=3}$$

$$\therefore -\frac{5}{4}x_2-\frac{3}{4}=-2$$

$$\therefore 5x_2+3=8$$

$$\therefore 5x_2=5$$

$$\therefore x_2=\underline{1}.$$

$$4x_3+3\cdot 1+3\cdot 3=20$$

$$\therefore 4x_3+3+9=20$$

$$\therefore 4x_3=20-12$$

$$\therefore x_3=\frac{8}{4}=2$$

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partial pivoting :

In the first stage of elimination, the first column is seached for the largest element in magnitude and brought as the first pivot by interchanging the first equation with the equation having the largest element in magnitude. In the second elimination stage, the second column is seached for the largest element in magnitude among $n-1$ elemets leaving the first element, and this elements is brough as the second pivot by an interchange of the second equation with the eqn having the largest element in magnitude. This procedure is contined until we arrive at the system coeffn at the upper triangular matrix. We are thus led to the following algorithm to find the pivot.

choose j, the smallest integer for which

$$|a_{jk}^{(k)}| = \max |a_{ik}^{(k)}|, \quad k \le i \le n.$$

and interchange row k and j

complete pivoting

we search the matrix A for the largest element in magnitude and bring it as the 1st pivot. This requires not only an interchange of equation but also an interchange of the position of the variables. This leads us to the following algorithm to find the pivat.

choose l and m as the smallest integers for which $|a_{lm}^{(k)}| = \max |a_{ij}^{(k)}|$, $k \le i, j \le n$.

and interchange row k and l and columns k and m.

HW:

Solve the system of equations

$$\begin{bmatrix} 2 & 1 & 1 & -2 \\ 4 & 0 & 2 & 1 \\ 3 & 2 & 2 & 0 \\ 1 & 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -10 \\ 8 \\ 7 \\ -5 \end{bmatrix}$$

using Gauss elimination method with partial and complete pivoting