Test for convergence and divergence for series of positive (or non-negative) Terms.

An Important property:-

The terms in a series of positive (or non-negative) terms can be grouped or rearranged in any way without affecting the character of the series and without changing the sum of a convergent series.

Note:- This property does not hold for a series of mixed terms as can be observed from this example. We consider the series $\sum_{n=1}^{\infty} u_n$ of mixed terms:-

$1-1+1-1+1-1+\cdots$

30	31					1	
2	3	4	5	6	7	8	JULY '17
9	10	11	12	13	14	15	
16	17	18	19	20	21	22	
23	24	25	26	27	28	29	
S	M	T	W	T	F	S	

The sequence S_n: 1, 0, 1, 0, 1, 0, ---

Oscillates and thus the series $\sum_{n=1}^{\infty} u_n$ is not convergent.

But if we group the series like

$(1-1) + (1-1) + (1-1) + ---$, it is equivalent to series: $0 + 0 + ---$, which is convergent as $S_n = 0 \ \forall n$.

Comparison Test :—

Statements :—

(i) If $\sum u_n$ and $\sum v_n$ be two series of positive terms, such that $u_n \le k v_n$, for all $n \geq m$ (k: a positive constant and m: a positive integer) and the series $\sum v_n$ is convergent, then the series $\sum u_n$ is also convergent.

(ii) If $\sum u_n$ and $\sum v_n$ be two series of positive terms, such that $u_n \geq k v_n$, for all $n \geq m$ (k: a positive integer) and the series $\sum v_n$ is divergent, then the series $\sum u_n$ is also divergent.