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System of Linear Algebraic Equation and Eigenvalue problems

Let $2x+3y+5z=1$

$5x+z=1$

$5y+2z=2$

This is called System of Linear Algebraic Equation in 3 unknown (x,y,z) and 3 eqn

This eqn can be written as

$AX=b$ where $A=\begin{pmatrix}2&3&5\\5&0&1\\0&5&2\end{pmatrix}$

$X=\begin{pmatrix}x\\y\\z\end{pmatrix}$ $b=\begin{pmatrix}1\\1\\2\end{pmatrix}$.

Consider a system of n linear algebraic eqn in n-unknown.

$$a_{11}x_1+a_{12}x_2+\cdots+a_{1n}x_n=b_1$$
$$a_{21}x_1+a_{22}x_2+\cdots+a_{2n}x_n=b_2$$
$$\vdots$$
$$a_{n1}x_1+a_{n2}x_2+\cdots+a_{nn}x_n=b_n$$

where a_{ij} $(i,j=1(1)n)$ are the know coefficients b_i, $(i=1(1)n)$ are known values and x_i, $(i=1(1)n)$ are the unknowns to be determined.

Then this system of eqn in n unknow and n eqn canbe written as matrix form

$Ax = b$ where $A = (a_{ij})_{n\times n}$ and $b = (b_i)_{1\times n}$
and $x = (x_i)_{1\times n}$
we can write
$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$
$$b = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$$
$$x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

Notation:

$A \rightarrow$ Squre matrix of order n.
$a_{ij} \rightarrow$ element in the ith row and j-th column of the matrix A
$A^{-1} \rightarrow$ inverse of A
$A^T \rightarrow$ transpose of A
$|A| \rightarrow$ determinant of A
$A_{ij} \rightarrow$ cofactor of a_{ij} in A.
$O \rightarrow$ null matrix.
$I \rightarrow$ identity matrix of or n.
$D \rightarrow$ Diagonal matrix of order n.
$L \rightarrow$ lower triangular matrix of order n.
$\|A\| \rightarrow$ norm of A

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Defination: Let A be a real matrix. Then A is said to be – ($A=(a_{ij})_{n\times n}$, $a_{ij}\in\mathbb{R}$).

(1) nonsingular: if $|A|\neq 0$.

(2) symmetric: if $A^T=A$

(3) skew-symmetric: if $A^T=-A$.

(4) orthogonal: if $AA^T=I$ or $A^T=A^{-1}$

(5) null: if $A=(a_{ij})_{n\times n}$

$a_{ij}=0 \quad i,j=1(1)n$.

(6) diagonal: if $a_{ij}=0,\ i\neq j$

(7) unit matrix: if $a_{ij}=0\ i\neq j \quad a_{ii}=1 \quad i=1(1)n$

(8) lower traingular: if $a_{ij}=0 \ j>i$

(9) upper traingular: if $a_{ij}=0 \ i>j$

10. diagonally dominant:

$$\text{if } |a_{ii}| \geq \sum_{\substack{j=1\\ i\neq j}}^{n} |a_{ij}| \quad i=1(1)n.$$

11. Let $A=(a_{ij})_{n\times n}$, $a_{ij}\in\mathbb{C}$ $i,j=1(1)n$. be a complex matrix –

Then A is called

(i) Hermitian: when $(\bar{A})^T=A$

(2) Skew ": when $(\bar{A})^T=-A$

(3) unitary if $A^{-1}=(\bar{A})^T=A^*, A^\theta$.

(4) normal if $AA^*=A^*A$.

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Linear System of Equation:

In the matrix notation, the previous system can be written as

$$Ax = b. \longrightarrow (1)$$

where A is called coeficient matrix.

Then $[A|b]$ is called the augmented matrix. It is formed by appending the column b to the $n\times n$ matrix A.

▨ If all b_i are zero (i.e, $b=0$) then the system (1) is called homogeneous system (i.e $Ax=0$) and if $b_i \neq 0$ for some $i = 1(1)n$ then the system (1) is called in-homogeneous or non-homogeneous system of eqn.

▨ (i) The inhomogeneous system (1) has a unique solution iff $|A| \neq 0$.

i.e $\det A = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a & - & - & - \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} \neq 0.$

The solution of the system (1) may be written as

$$X = A^{-1} b.$$

if the system is homogeneous and $|A| \neq 0$ then the only soln is $X = 0$.

▨ The methods of solution of the linear algebraic eqn (1) may be classified into two types.

(i) Direct method: (Cramer Rule / triangular meth)

(ii) Iterative method: (Gauss Elimination method