Here $\alpha \in S \cup T$, $\beta \in S \cup T$

but $\alpha+\beta \notin S$, $\alpha+\beta \notin T$

$\therefore$ $\alpha+\beta \notin S \cup T$

Hence $S \cup T$ is not a subspace of $\mathbb{R}^3$.

*) Determine the subspace of $\mathbb{R}^3$ spanned by the vector $\alpha = (1,2,3)$, $\beta = (3,1,0)$. Examine if (i) $\gamma = (2,1,3)$ is in the subspace
(ii) $\delta = (-1,3,6)$ is in the subspace

$\rightarrow$ $L\{\alpha, \beta\}$ is the set of vectors $\{c\alpha + d\beta : c \in \mathbb{R}, d \in \mathbb{R}\}$

$\therefore c\alpha + d\beta = c(1,2,3) + d(3,1,0)$
$= (c+3d, 2c+d, 3c)$

If $\gamma \in L\{\alpha, \beta\}$ then there must be real numbers c, d such that

$(2,1,3) = (c+3d, 2c+d, 3c)$

$\therefore$ $c+3d = 2$, $2c+d = 1$, $3c = 3 \Rightarrow c = 1$

These eq^n are in consistent and so γ is not in $L\{\alpha, \beta\}$.

If $\delta \in L\{\alpha, \beta\}$ then there must be real numbers c, d such that

$(-1, 3, 6) = (c + 3d, 2c + d, 3c)$

Therefore, $c + 3d = -1$, $2c + d = 3$, $3c = 6$

giving $c = 2, d = -1$

Therefore $\delta = 2(1, 2, 3) - 1(3, 1, 0)$

Showing that $\delta \in L\{\alpha, \beta\}$,

(**) Let $S = \{\alpha, \beta, \gamma\}$, $T = \{\alpha, \beta, \alpha+\beta, \beta+\gamma\}$ be subsets of a real vector space V

Show that $L(S) = L(T)$

⇒ Each elements of T is a linear combination of the vectors S.

$\therefore L(T) \subset L(S)$ ---- (1)

Again $\alpha = \alpha + 0\beta + 0(\alpha+\beta) + 0(\beta+\gamma)$,

$\beta = 0\alpha + \beta + 0(\alpha+\beta) + 0(\beta+\gamma)$

$\gamma = 0\alpha - \beta + 0(\alpha+\beta) + (\beta+\gamma)$

This shows that each element of S is a linear combination of the vectors of T. Therefor $L(S) \subset L(T)$

$\therefore L(S) = L(T)$.

Linear dependence and linear independence.

⇒ A finite set of vectors $\{\alpha_1, \alpha_2, \dots, \alpha_n\}$ of a vector space V over a field F is said to be linearly dependent in V if there exists scalars $c_1, c_2, \dots, c_n$ not all zero in F such that

$$c_1\alpha_1 + c_2\alpha_2 + \dots + c_n\alpha_n = \theta \quad \text{--- (i)}$$

The set is said to be linearly independent in V if the equality (i) is satisfied only when

$$c_1 = c_2 = \dots = c_n = 0$$

Ex:- Prove that the set of vectors $\{(1,2,2), (2,1,2), (2,2,1)\}$ is linearly independent in $\mathbb{R}^3$.

Let $\alpha = (1,2,2)$, $\beta = (2,1,2)$, $\gamma = (2,2,1)$

Let us consider a relation

$$c_1\alpha + c_2\beta + c_3\gamma = \theta \quad \text{where } c_1, c_2, c_3 \text{ are real no.}$$

Then $c_1(1,2,2) + c_2(2,1,2) + c_3(2,2,1) = (0,0,0)$

$\therefore c_1 + 2c_2 + 2c_3 = 0$, $2c_1 + c_2 + 2c_3 = 0$ and $2c_1 + 2c_2 + c_3 = 0$.

Now $\begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix} \neq 0$ ∴ by Cramer's rule there exists a unique solⁿ, and the solⁿ is $c_1 = 0, c_2 = 0, c_3 = 0$.

∴ the set of vector are linear indep.

27 FRIDAY JAN
Week 04

Quotient Space:— Let W be a vector Sub-Space of V and let V/W denote the set of cosets of V with respect to W. Then the operations $\alpha_1+\alpha_2=(\beta_1+\beta_2)+W$, where $\alpha_i=\beta_i+W\in V/W$ and $a\alpha=(a\beta)+W$ where $a\in R$ and $\alpha=\beta+W\in V/W$ are well defined and with these operations, V/W becomes a vector space. This vector space V/W is said to be the quotient space of V with respect to W.

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				
S	M	T	W	T	F	S

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Definition of Basis:–

Let V be a vector space over a field F. A set S of vectors in V is said to be a basis of V if

(i) S is linearly independent in V.

(ii) S generates V ie $L(S) = V$.

Ex: The set $E = \{ \epsilon_1 = (1,0), \epsilon_2 = (0,1) \}$ is a basis of $\mathbb{R}^2$.

To prove the linear independence of the set E.

Let us consider a relation

$c_1\epsilon_1 + c_2\epsilon_2 = \theta$ where $c_1, c_2 \in \mathbb{R}$

ie $c_1(1,0) + c_2(0,1) = (0,0)$

$\therefore c_1 = c_2 = 0$

$\therefore E$ is linearly independent

Let $\xi = (a,b)$ be an arbitrary vector of $\mathbb{R}^2$

Let us examine $\xi \in L(E)$ if possible let

$\xi = r_1\epsilon_1 + r_2\epsilon_2$ with $r_1, r_2 \in \mathbb{R}$

ie $(a,b) = r_1(1,0) + r_2(0,1)$

$\therefore r_1 = a, r_2 = b.$

$\therefore \xi \in L(E)$

		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28					
M	T	W	T	F	S	S

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Therefore $\mathbb{R}^2 \subset L(E)$

Again $E \subset \mathbb{R}^2$ and $L(E)$ being the smallest subspace containing E

$\therefore L(E) \subset \mathbb{R}^2$

$\therefore \quad L(E) = \mathbb{R}^2$

$\therefore E$ is a basis of $\mathbb{R}^2$.

Basis and dimension of a vector space:—
Let V be a vector space over the field F and S be a sub-set of V(F) such that (i) S is a set of linearly independent vectors in V and (ii) $L(S) = V$, that is, each vector in V is a linear combination of a finite number of elements of S, (S generates V) then S is called a basis set or simply a basis of V.

A zero vector cannot be an element of a basis set, because such a set is linearly dependent.

Consider the set $B = \{(1,0,0), (0,1,0), (0,0,1)\}$ in V_3 over the real numbers

This set is linearly independent.
Also B spans V_3, because any vector (a_1, a_2, a_3) of V_3 can be written as a linear combination of the vectors of B for example,

$$(a_1, a_2, a_3) = a_1(1,0,0) + a_2(0,1,0) + a_3(0,0,1)$$

Hence B is a basis of V_3 over the real numbers.

It is called the standard basis of R^3

Consider, again, the set $B' = \{(1,1,0), (1,0,1), (0,1,1)\}$ in V_3 over the real numbers.

The relation $a(1,1,0) + b(1,0,1) + c(0,1,1) = 0 = (0,0,0)$ implies $a = b = c = 0$,

that is B' is linearly independent.

Furthermore, any vector (a_1, a_2, a_3) of V_3 can be written as a linear combination of the vectors of B' such as

$$(a_1, a_2, a_3) = \frac{1}{2}(a_1 + a_2 - a_3)(1,1,0)$$
$$+ \frac{1}{2}(a_1 + a_3 - a_2)(1,0,1)$$
$$+ \frac{1}{2}(a_2 + a_3 - a_1)(0,1,1)$$

Thus B' is also a basis of V_3 over the real numbers.

MARCH '17			1	2	3	4	5
	6	7	8	9	10	11	12
	13	14	15	16	17	18	19
	20	21	22	23	24	25	26
	27	28	29	30	31		
	M	T	W	T	F	S	S